Multi-Set Multi-Outcomes Games

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**Introduction**

This paper deals with something called a multi-set multi-outcome game. It is a set of cooperative games where although equilibrium may be found, looking ahead can disturb that equilibrium. A simple equilibrium may find itself different after several iterations of the look-ahead process. These games are meant to represent the problem: “Should I be forced to make the choice in a game, without knowing the entire future of my choice, should I still make it or select another.”

**Example 1**

Suppose you had the following game:

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 1,1 | 0,1 |
| B | 1,0 | 0,0 |

Typically in a cooperative game AC would be the best choice. However…

Suppose the lookahead of the game is:

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 0,0 | 1,0 |
| B | 0,1 | 1,1 |

Then the outcome of AC with respect to the rest of the game has changed drastically. Now the game is:

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 1,1 | 1,1 |
| B | 1,1 | 1,1 |

In fact now we see, if we look into the future in our game, it turns out that eventually any choice we make will result in the same gain/loss.

**Example 2**

Suppose we have the following game:

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 0,0 | 0,0 |
| B | 1,0 | 1,1 |

Here it is clear BD is the winner.

But looking ahead we find

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 0,0 | 2,0 |
| B | 1,1 | 0,0 |

Here the best choice is (cooperative) BC. However, summing the lookahead with the original table as we have been doing shows

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 0,0 | 2,0 |
| B | 2,1 | 1,1 |

In this case, BC is the winning choice.

Now we will attempt to make a connection to graph theory. I believe game theory can be used to solve the traveling salesman problem by sending out different players that act as travelling salesmen, and see who wins.

But I digress.

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | a,b 16 | c,d 2 |
| B | a,d 2 | b,e 4 |

Here obviously the winner for largest choice is AC. Let’s see if we can do better if we look ahead.

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | b,x 1 | c,d 2 |
| B | a,d 2 | b,e 4 |

Here the AC strategy is still dominant. Let us lookahead one more time.

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | c,d 12 | d,e 15 |
| B | b,d 15 | e,f 12 |

So the final strategy is

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 29 | 19 |
| B | 19 | 20 |

So overall, the strategy AC dominates, working like an exponential function, continually growing.